Scattering from Ferrite Bodies of Revolution Using a Hybrid Approach

Larry W. Epp, *Member, IEEE*, Daniel J. Hoppe, *Member, IEEE*, and Gilbert C. Chinn, *Student Member, IEEE*

Abstract— **The scattered fields from axisymmetric problems containing anisotropic media are found by a Hybrid Finite Element method. In particular a symmetric formulation for bodies of revolution that incorporates a Finite Element formulation for axially magnetized ferrite materials is presented. The method is applied to a ferrite cylinder with quartz matching layers. A Gaussian beam input is used to predict the Faraday rotation through the ferrite cylinder and display it visually.**

I. Introduction

More commonly known for their use in waveguide devices such as circulators, phase shifters, and isolators, ferrite materials also have antenna applications. Radiation from antennas in the presence of ferrites gives rise to interesting devices that take advantage of the non-reciprocal properties of ferrite materials, [1], [2].

Recently it has been shown that the Finite Element method, in conjunction with the use of edge elements, can be extended to ferrite materials [3], [4]. The purpose of this work is to show how this Finite Element approach can be incorporated into a hybrid integral equation/FEM method [5]. The modifications for the internal region for an axially magnetized ferrite are considered explicitly. The problem of scattering by a ferrite cylinder with quartz matching layers is chosen to demonstrate the method.

II. Description of Method

The formulation begins with the vector wave equation for the electric field in an anisotropic, inhomogeneous region with the constitutive relations given by, $\vec{\mathbf{B}} = \mu_0 \overline{\mu}_r \cdot \vec{\mathbf{H}}$, and $\vec{\mathbf{D}} = \epsilon_0 \epsilon_r \vec{\mathbf{E}}$. The relative permeability tensor characterizing an axially biased ferrite is given by [6]

$$\overline{\mu}_r = \begin{pmatrix} \mu & -j\mu' & 0 \\ j\mu' & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix}. \tag{1}$$

The sign of μ' indicates that the bias field is in the $\hat{\mathbf{a}}_z$ direction.

Manuscript received September 23, 1994. This work was supported by the JPL Supercomputing Project (sponsored by the Jet Propulsion Laboratory and the NASA Office of Space Sciences and Applications).

The authors are with the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 USA.

IEEE Log Number 9409013.

The method of weighted residuals is used to solve for the electric field

$$\int_\Omega \left(\nabla \times \vec{\mathbf{E}}^*\right) \cdot \left(\overline{\mu}_r{}^{-1} \cdot \nabla \times \vec{\mathbf{E}}\right) d\Omega - \int_\Omega k_0^2 \epsilon_r \vec{\mathbf{E}}^* \cdot \vec{\mathbf{E}}\, d\Omega - j\omega\mu_0 \int_S \vec{\mathbf{E}}^* \cdot \left(\hat{\mathbf{n}} \times \vec{\mathbf{H}}\right) dS = 0. \tag{2}$$

In the present development, the testing or weighting functions $\vec{\mathbf{E}}^*$ are chosen to be the conjugate of the basis functions.

A surface integral is used to truncate the finite element region in the hybrid symmetric formulation of [5]. In the interior Finite Element region the electric field is broken into vectors transverse and normal to ϕ. The matrix element Z^{EE} as given in [5] results from the volume integral in (2):

$$Z_{\alpha\beta}^{EE}(i, j) = S_{\alpha\beta}(i, j) + T_{\alpha\beta}(i, j) \tag{3}$$

with α and β representing either the transverse, t, or the ϕ components.

The inclusion of the anisotropic material modifies $S_{\alpha\beta}(i, j)$ but $T_{\alpha\beta}(i, j)$ remains unchanged. The tensor permeability contributes to additional matrix fill of the S_{tt}, $S_{\phi t}$, and $S_{t\phi}$ components due to the presence of a finite μ'. A more detailed expansion of $S_{\alpha\beta}(i, j)$ shows that the formulation remains symmetric for a lossless permeability tensor as given by (1).

III. Numerical Results

A. Ferrite Cylinder with Quartz Matching Layers, Plane Wave Excitation

Consider the ferrite cylinder with quartz matching layers as depicted in Fig. 1. This device, when subject to an external biasing field, is similar to the quasi-optical Faraday rotator of Dionne *et al.* [7]. Following [7] the quartz matching layers have been chosen to be approximately a quarter-wavelength thick at $f = 17.15$ GHz. This provides an impedance match to free space since the relative dielectric constant of quartz, ϵ_q, approximately satisfies the condition $\epsilon_q \approx \sqrt{\epsilon_f}$, where ϵ_f is the relative dielectric constant of the ferrite.

Plane wave scattering by the ferrite cylinder with and without a biasing magnetic field was considered. For the unbiased case comparison is made with an integral equation solution based on the PMCHW [8] formulation. In this case

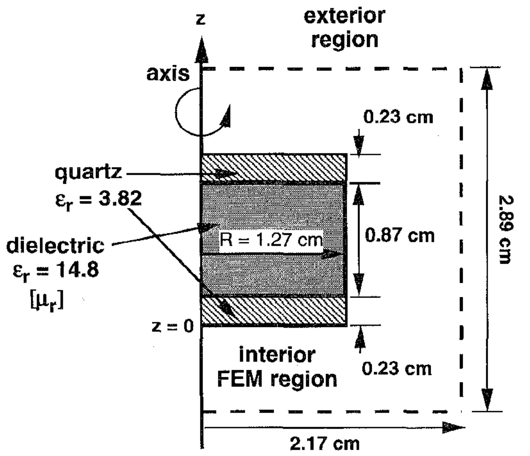

Fig. 1. Geometry of an inhomogeneous, anisotropic cylinder.

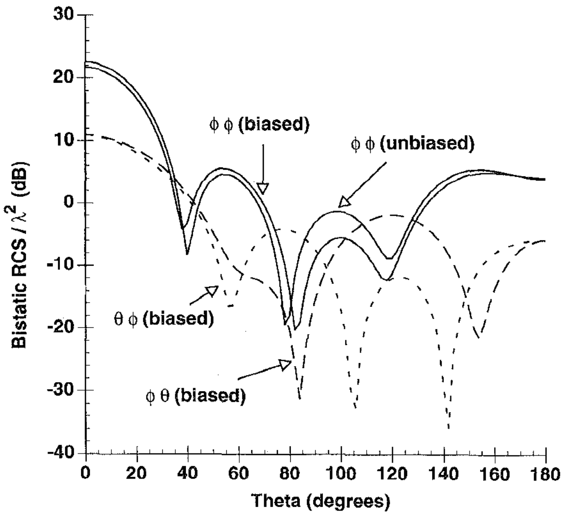

Fig. 2. Plot of the bistatic $\phi\phi$, $\phi\theta$, and $\theta\phi$, RCS of a dielectric/ferrite cylinder ($\epsilon_f = 14.8$) with quartz ($\epsilon_r = 3.82$) matching layers, $f = 17.15$ GHz for the biased and unbiased cases.

the integral equation solution consists of 3332 unknowns placed on the exterior of the ferrite/quartz cylinder and in the interfaces between the ferrite and the quartz. In both the biased and unbiased cases the Finite Element mesh consisted of 18 639 triangular elements in the ferrite, quartz, and free space regions. The external integral equation boundary was applied at a radius of $R = 2.17$ cm as shown in Fig. 1, and a total of 139 triangles were placed on the integral equation boundary. The matrix order consisting of FEM and integral equation unknowns was 37 455 for the $n = \pm 1$ harmonics. Typical run times for the results of this paper are 2 hours on a Cray C98.

Fig. 2 shows the computed bistatic $\phi\phi$ RCS, based on the methods described above for the unbiased and biased ferrite cases, as well as the $\phi\theta$ RCS and $\theta\phi$ RCS for the biased case. If α is either θ or ϕ then $RCS_{\alpha\beta}$ is computed from the value of E_α for $\phi = 0°$ when the excitation is a plane wave polarized in the $E_\beta = E_\phi$ ($\hat{\mathbf{a}}_y$ direction) or $E_\beta = E_\theta$ ($\hat{\mathbf{a}}_x$ direction). The plane wave is incident from $\theta = 180.0°$ on the cylinder depicted in Fig. 1. For the unbiased case, where no external magnetic field would be applied, the relative permeability parameters are $\mu = 1.0$, $\mu_z = 1.0$, $\mu' = 0.0$, and the dielectric constant is $\epsilon_r = 14.8$. For the biased ferrite case for which $\mu = 0.98$, $\mu_z = 1.0$, $\mu' = -0.13$ the $\phi\phi$ RCS is only slightly modified from the unbiased case, as shown in Fig. 2. Results for the $\theta\theta$ RCS are similar. In all of the unbiased cases excellent agreement with the integral equation solution was observed. More indicative of the biased ferrite are the RCS calculations of $\theta\phi$ and $\phi\theta$ also shown in Fig. 2, which are identically zero in the unbiased case.

B. Ferrite Cylinder with Quartz Matching Layers, Gaussian Beam Excitation

For validation of the anisotropic case, an excitation is needed that allows comparison with the infinite ferrite media. Such an excitation should simulate the constant phase along the front surface of the cylinder of the plane wave. It is also desirable that the magnitude of the field components decrease significantly at the outer radius of the cylinder in order to reduce edge effects. Such an incident wave is a Gaussian beam, which is a solution of the wave equation subject to the paraxial approximation.

Computations were carried out for a Gaussian beam, polarized in the $\hat{\mathbf{a}}_y$ direction incident from $\theta = 180°$ on the cylinder depicted in Fig. 1. The relative permeability/permittivity values of the ferrite are as given above for the plane wave case. The frequency is $f = 17.15$ GHz. The beam waist of the incident Gaussian field was 1.1 cm and was placed at the first air to quartz interface, or $z = 0$.

The rotation angle for the transmitted wave along the z axis in the far field was found to be $-44.69°$. The wave was found to have an axial ratio of 4.29, indicating some cross polarized component in the transmitted field. Plane wave analysis of an infinite ferrite media with the parameters of $\mu = 0.98$, $\mu_z = 1.0$, $\mu' = -0.13$ and $\epsilon_r = 14.8$ gives a rotation angle of $\theta_f = -45.33°$. Applications of such a device include recent phase locking of a high power pulsed gyrotron oscillator [9].

The observed ellipticity may be partly attributed to different impedance matches caused by the distinct phase constants of the RCP and LCP waves inside the ferrite material, and the imperfect impedance match provided by the quartz layers. A more significant factor is the high edge illumination on the disk, (-11.6 dB). This causes a diffracted field which contributes to the cross polarization. This was confirmed by reducing and increasing the waist size of the incident beam and examining the axial ratio of the transmitted beam. As expected lower edge illumination produced results which were in closer agreement with the plane wave analysis of an infinite media.

Another method of verifying the Faraday rotation is to look at the direction of the electric field as it progresses through the ferrite cylinder. Fig. 3 shows the magnitude of the electric field projected onto the x and y planes for the beam waist of 1.1 cm. Thus on the x plane the representation is of $|\vec{\mathbf{E}} \cdot \hat{\mathbf{a}}_x|$, and likewise for the y plane. Fig. 3 clearly shows the Faraday rotation for the $\hat{\mathbf{a}}_y$ polarized incident Gaussian field. On the incident side the projections of the field show mainly on the y plane. But on exit the magnitudes are split equally between the two planes, indicative of the $-44.69°$ rotation.

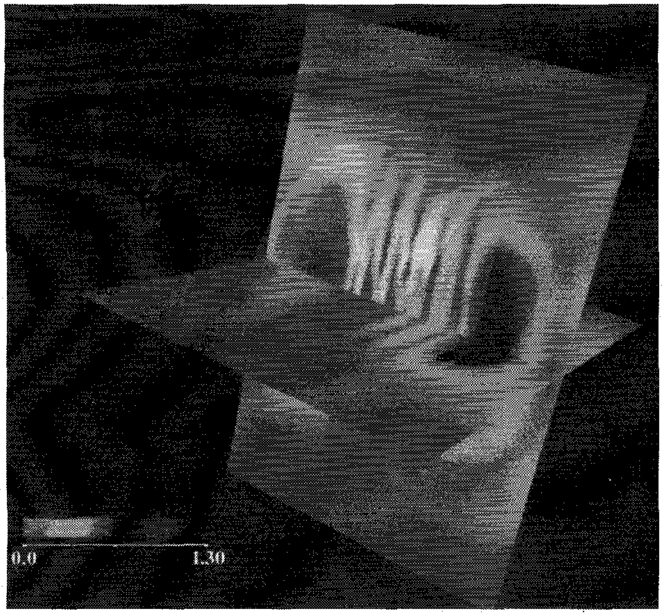

Fig. 3. Projection of the magnitude of the electric field along the $\hat{\mathbf{a}}_x$ and $\hat{\mathbf{a}}_y$ directions for a Gaussian beam ($\omega_0 = 1.1$ cm) incident on the ferrite cylinder ($\epsilon_r = 14.8$, $\mu = 0.98$, $\mu' = -0.13$) of Fig. 1.

IV. Conclusion

A Hybrid Finite Element method for axially magnetized ferrite materials has been developed. Application of this method to predict the scattering from a ferrite cylinder was presented for plane wave incidence as well as for Gaussian beam incidence and displayed visually. The calculated rotation angle of the major axis of the transmitted field was used as a point of comparison with the rotation of a plane wave in an infinite ferrite medium.

References

[1] D. M. Pozar, "Radiation and scattering characteristics of microstrip antennas on normally biased ferrites," *IEEE Trans. Antenn. Propagat.*, vol. 40, no. 9, pp. 1084–1092, Sept. 1992.

[2] N. E. Buris, T. B. Funk, and R. S. Silverstein, "Dipole arrays printed on ferrite substrates," *IEEE Trans. Antenn. Propagat.*, vol. 41, no. 2, pp. 165–176, Feb. 1993.

[3] J. S. Wang and N. Ida, "Eigenvalue analysis in anisotropically loaded electromagnetic cavities using 'edge' finite elements," *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1438–1441, Mar. 1992.

[4] B. M. Dillon, A. A. P. Gibson, and J. P. Webb, "Cut-off and phase constants of partially filled axially magnetized, gyromagnetic waveguides using finite elements," *IEEE Trans. Microwave Theory Tech.*, vol. 41, no. 5, pp. 1438–1441, May 1993.

[5] D. J. Hoppe, L. W. Epp, and J. F. Lee, "A hybrid symmetric FEM/MOM formulation applied to scattering by inhomogeneous bodies of revolution," *IEEE Trans. Antenn. Propagat.*, vol. 42, no. 6, pp. 798–805, June 1994.

[6] B. Lax and K. J. Button, *Microwave Ferrites and Ferrimagnetics.* New York: McGraw-Hill, 1962.

[7] G. F. Dionne, J. A. Weiss, G. A. Allen, and W. D. Fitzgerald, "Quasi-optical ferrite rotator for millimeter waves," in *IEEE MTT Symp. Dig.*, pp. 127–130, 1988.

[8] P. L. Huddleston, L. N. Medgyesi-Mitschang, and J. M. Putnam, "Combined field integral equation formulation for scattering by dielectrically coated conducting bodies," *IEEE Trans. Antenn. Propagat.*, vol. AP-34, pp. 510–520, Apr., 1986.

[9] D. J. Hoppe, R. M. Perez, H. Z. Guo, J. Rogers, and J. P. Tate, "Phase locking of a second harmonic gyrotron using a quasi-optical circulator," submitted to the International Society for Optical Engineering's *Int. Symp. Intense Microwave Pulses III.*